CALCULATION OF INDUCTIVE ELECTRIC FIELDS IN PULSED COAXIAL DEVICES USING ELECTRIC VECTOR POTENTIALS

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Abstract

In many high-voltage pulsed power systems the electric fields are predominantly inductive rather than electrostatic. That is, in the usual expression for generalized electric field, $\mathbf{E} = -\nabla\Phi - \partial\mathbf{A}/\partial t$, Φ is the scalar potential that gives rise to the electrostatic field, and $\mathbf{A}$ is the magnetic vector potential, from which the inductive field is derived. In problems where there are regions without charge separation or steady state currents flowing, the electrostatic component does not exist, and the usual technique of solving the scalar Laplace's equation for the potential is inappropriate for determining the electric fields. Calculation of the magnetic vector potential is plagued by choice of gauge condition and specification of correct boundary conditions. Especially for coaxial (axisymmetric) systems typical of many pulsed power components and systems, where the current flow is in the r,z plane, there are two components of the vector potential that must be solved—each with its own boundary conditions. Specification of all the correct boundary conditions is non-trivial.

In this paper, we present a convenient technique for the calculation of inductive electric fields in coaxial systems. The technique is based on the introduction of a vector electric potential that is derived from Poisson's equation, in combination with Faraday's Law and the $\boldsymbol{E}$, $\boldsymbol{D}$ constitutive relation. In coaxial geometry, the electric vector potential is only azimuthal and, therefore, quasi-scalar. It is conveniently calculated with any two-dimensional Poisson equation solver, and the resultant inductive field distribution easily calculated. We have used the technique in several pulsed power system designs with success. Specific examples of the application of the technique are given.

I. INTRODUCTION

In high-voltage pulsed power apparatus design, one must always be concerned with electrical breakdown that arises when electric field exceeds breakdown thresholds. In electrostatic systems, the field is given solely by the gradient of a scalar potential,

$$\mathbf{E}_{es} = -\nabla\Phi\ . \qquad \textbf{(1)}$$

For dynamic pulsed systems, the electric fields may be primarily inductive—that is, due to time-varying magnetic fields associated with time-varying currents. It can be shown that, in general, if $\mathbf{A}$ is the magnetic vector potential ($\mathbf{B} = \nabla\times\mathbf{A}$), then the inductive electric field component is given by

$$\mathbf{E}_{ind} = -\frac{\partial\mathbf{A}}{\partial t}\ . \qquad \textbf{(2)}$$

For axisymmetric configurations with current flow only in the azimuthal (θ) direction, the magnetic vector potential has only one component (θ), also, so that when Eq. (2) is substituted into Ampere's Law, a single scalar equation results. As in the case for two-dimensional electrostatic field calculation from Laplace's Equation, any of several two-dimensional partial differential equation solvers can be used to calculate the potentials, and from them the fields.

However, when the magnetic fields are in the azimuthal direction (as for coaxial current flow configurations, typical of many pulsed power devices), the magnetic vector potential generally has two components, A_r and A_z. Solution now is no longer straightforward. In order to solve for the inductive electric field in such cases, we have developed a new technique that results in a single quasi-scalar potential-type equation that is relatively easy to solve using standard finite-element partial differential equation programs.

In the next section, we present the mathematical formalism associated with the electric vector potential and discuss boundary conditions. We give an electric vector potential example in section III.

II. ELECTRIC VECTOR POTENTIAL

A. Basic Formulation

We begin with Poisson's equation in the absence of space charge,

$$\nabla\cdot\mathbf{D} = 0\ . \qquad \textbf{(3)}$$

Knowing that if the divergence of a vector is identically equal to zero then it can be represented as the curl of another vector, we express the electric flux density as the curl of an electric vector potential, $\mathbf{F}$,

$$\mathbf{D} = -\nabla\times\mathbf{F}\ . \qquad \textbf{(4)}$$

We now express the electric field in terms of the electric flux density, using the constitutive relation

$$\mathbf{D} = \varepsilon\,\mathbf{E}\,, \qquad \textbf{(5)}$$

$$\mathbf{E} = -\nabla \times \mathbf{F} / \varepsilon\,. \qquad \textbf{(6)}$$

Next, we use Faraday's Law,

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}\,, \qquad \textbf{(7)}$$

substituting for **E** from Eq. (6), to obtain finally

$$\nabla \times \left(\nabla \times \mathbf{F} / \varepsilon\right) = \frac{\partial \mathbf{B}}{\partial t}\,. \qquad \textbf{(8)}$$

Since, for axisymmetric azimuthal magnetic fields, the value of the magnetic flux density is given simply by

$$\mathbf{B} = \frac{\mu_0 I}{2\pi\, r}\hat{\theta}\,, \qquad \textbf{(9)}$$

the source term in Eq. (8) is usually known.

If the permittivity is uniform, at least in sub regions, then we can invoke a vector identity and rewrite Eq. (8) as

$$\nabla^2(\mathbf{F}) - \nabla(\nabla \cdot \mathbf{F}) = -\varepsilon \frac{\partial \mathbf{B}}{\partial t}\,. \qquad \textbf{(10)}$$

To simplify the calculation, we choose a "Coulomb-type" gauge condition for **F**; that is,

$$\nabla \cdot \mathbf{F} = 0\,, \qquad \textbf{(11)}$$

so that we obtain, finally, a vector Poisson equation for the electric vector potential,

$$\nabla^2(\mathbf{F}) = -\varepsilon \frac{\partial \mathbf{B}}{\partial t}\,. \qquad \textbf{(12)}$$

Note that both **F** and **B** have a θ component, only, so that the equation is quasi-scalar.

B. Boundary Conditions

Solution of Eq. (12) over a region requires specification of boundary conditions. Such boundary conditions can either be in terms of the value of the electric vector potential or its normal derivative (tangential electric flux density). In many cases, (*i.e.,* for good inductors) the conductor surface conductivity is sufficiently high that the tangential electric field is approximately zero in comparison to other boundary segments where the tangential electric field is known to be large.

Another boundary condition that often arises is that associated with an applied voltage source. In this case, one can usually specify a constant electric vector potential, so that the normal component of electric field (tangential derivative) is zero.

To obtain more accurate values of the tangential electric fields, or flux densities, one can solve the diffusion equation for the magnetic field, and from the solution calculate the tangential current density at the surface. The product of the surface current density and the conductor resistivity (inverse electrical conductivity) must match the tangential electric field at the surface. If the current density action integral,

$$J = \int j^2 dt\,, \qquad \textbf{(13)}$$

is sufficiently large, nonlinear effects may result from Joule heating and the subsequent temperature rise and conductivity decrease[1].

III. EXAMPLE CALCULATION

To illustrate the use of the electric vector potential, we present an example of the calculation of the inductive electric field inside a coaxial inductive cavity, to which a voltage source is applied. The geometry is shown in Fig. 1. In reality, the current around the cavity surface will be limited by the effective series resistance over a time scale approximately equal to the cavity inductance divided by the effective series resistance. For times short compared to this time scale, the voltage drop around the cavity is essentially inductive, and the developed formalism applies. The calculation tool we use to obtain the solution for both the vector potential and inductive electric field is FlexPDE™, a generalized two-dimensional finite element partial differential equation solver [2].

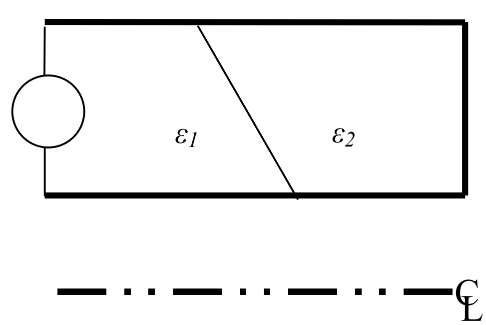

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Figure 1. Geometry for simple example.

In this example, we assume the cavity is filled with two different dielectric materials, with $\varepsilon_1 > \varepsilon_2$. The boundary condition around the surface of the cavity is that the normal derivative of **F** is approximately zero, $\nabla_\perp \times \mathbf{F} \approx \mathbf{0}$ (good conductor), and along the left edge, where the voltage source is applied, that the electric vector potential is zero, $\mathbf{F} = \mathbf{0}$ (no normal component of electric field).

Electric vector equipotential contours are shown in Fig. 2. Fig. 3 shows inductive electric field streamlines, and Fig. 4 is a contour plot of the magnitude of the electric field. Note the discontinuity in field lines from region 1 to region 2. By solving for the vector potential via finite element techniques, we automatically satisfy the interface conditions on **E** and **D**.

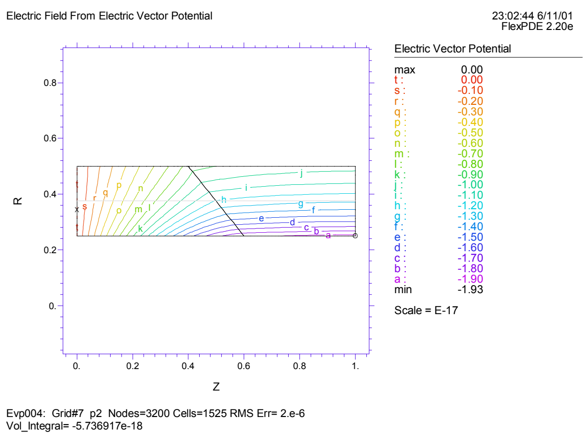

Figure 2. Electric vector equipotentials.

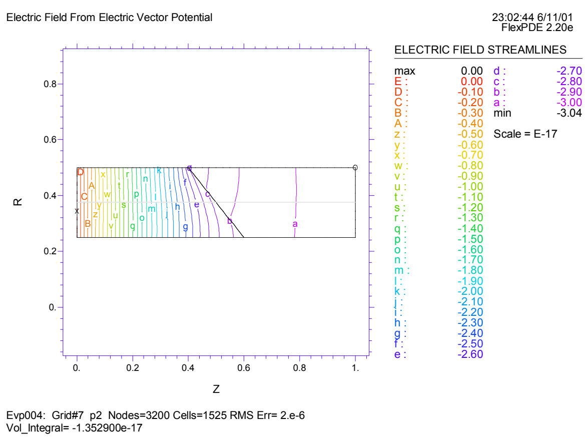

Figure 3. Inductive electric field streamlines.

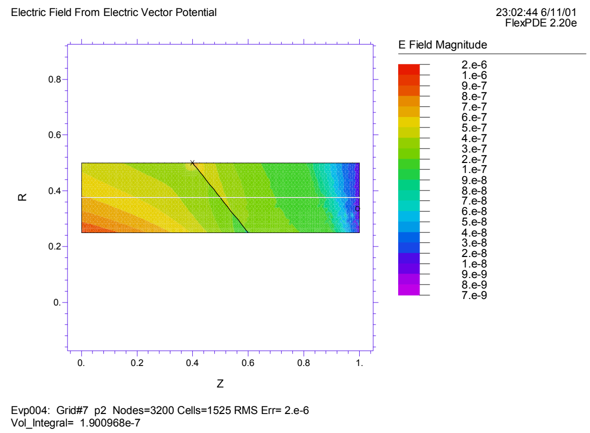

Figure 4. Inductive electric field magnitude.

IV. SUMMARY

In this paper we have presented a technique for calculating inductive electric fields associated with changing azimuthal magnetic fields in axisymmetric configurations. The technique is based on an electric vector potential, which only has an azimuthal component for this class of problems. The vector Poisson equation for the vector potential therefore is quasi-scalar, and it can be solved using any two-dimensional partial differential equation solver that can handle axisymmetry.

We have discussed the formalism for the electric vector potential and the boundary conditions required for its solution, from which the inductive electric field derives. We have demonstrated the technique with a simple example that illustrates the salient features.

The electric vector potential technique has been applied to the design of multiple pulsed power experiments and devices at AFRL, including high voltage pulse-forming elements in inductive storage circuits [3] and solid liner current feeds for NTLX experiments driven by the SHIVA STAR capacitor bank facility [4].

V. REFERENCES

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