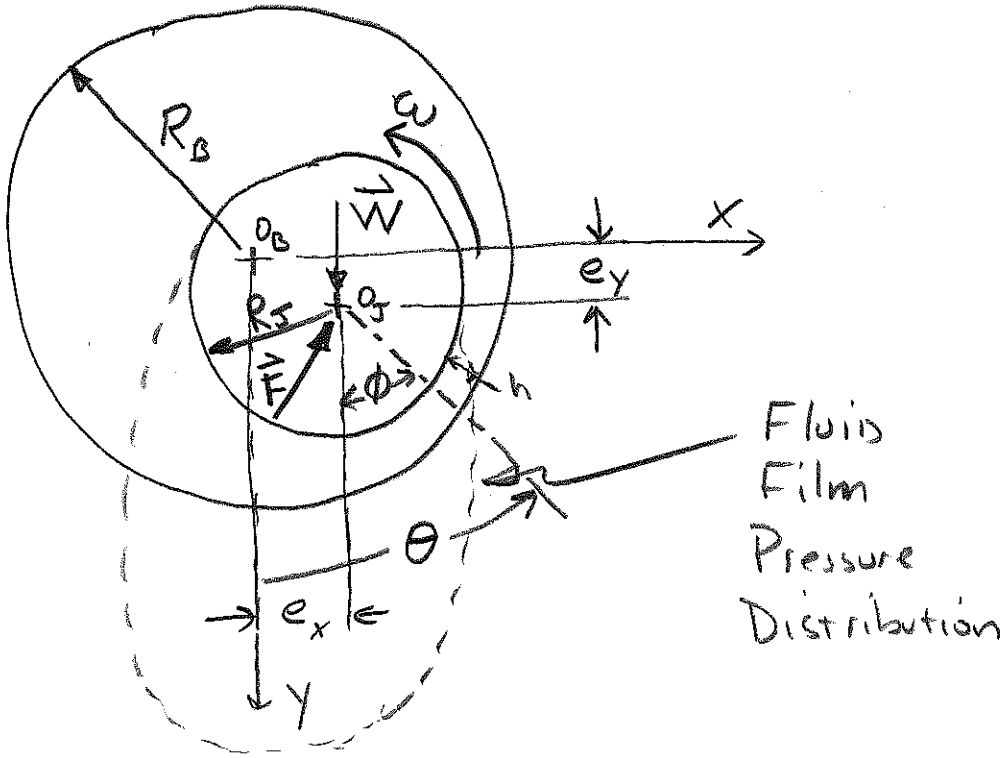
Generic Journal Bearing



$e_x = x_J - x_B$
$e_y = y_J - y_B$
} Journal Eccentricity

$C = R_B - R_A$

$C << R \equiv \dfrac{R_B + R_J}{2}$

$z \equiv R\theta$

L is bearing width (axial)
R_B is bearing radius
R_J is journal (shaft) radius
μ is viscosity
$p(z, z)$ is the fluid film pressure Distribution
$h(z, z)$ is the film thickness

Reynolds Lub. Eqn. (RLE)

$$\frac{\partial}{\partial z}\left(\frac{h^3}{\mu}\left(\frac{\partial p}{\partial z}\right)\right) + \frac{\partial}{\partial z}\left(\frac{h^3}{\mu}\left(\frac{\partial p}{\partial z}\right)\right) = 6\omega R\frac{\partial h}{\partial z} + 12\frac{\partial h}{\partial t}$$

where $p = p(z, z)$, $h = h(z, z)$, $0 \le z \le 2\pi R$

$-L/2 \le z \le L/2$

To Solve RLE for this application:

① <u>Specify</u> $e \equiv \sqrt{e_x^2 + e_y^2}$, $\phi = \tan^{-1}\left(\frac{e_y}{e_x}\right)$

$$h = C - e_x \cos(\tau/R) - e_y \sin(\tau/R)$$

$$\frac{\partial h}{\partial \tau} = \frac{e_x}{R}\sin\left(\frac{\tau}{R}\right) - \frac{e_y}{R}\cos\left(\frac{\tau}{R}\right)$$

$$\dot{h} = -\dot{e}_x \cos\left(\frac{\tau}{R}\right) - \dot{e}_y \sin\left(\frac{\tau}{R}\right)$$

② Solve RLE for $p = p(\tau, z)$

To get force in x & y directions

③ Integrate $p(\tau, z)$ over journal surface to get x and y forces

$$F_x = -\int_{-L/2}^{L/2}\int_0^{2\pi R} p(\tau, z)\cos\left(\frac{\tau}{R}\right) d\tau\, dz$$

$$F_y = -\int_{-L/2}^{L/2}\int_0^{2\pi R} p(\tau, z)\sin\left(\frac{\tau}{R}\right) d\tau\, dz$$

④ Calculate resultant radial load & angle

$$W = \sqrt{F_x^2 + F_y^2} \qquad \theta_W = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

③ Solve RLE over a range of values for

$$0 \leq \frac{e}{c} < 1 \quad \text{and} \quad \phi$$

Solutions to the RLE will be nonlinear for a journal bearing so the F_x & F_y will be nonlinear. The coefficients needed to model the stiffness & dampening of the bearing are given as:

$$K_{ij} = -\frac{\partial F_i}{\partial x_j}$$

$$C_{ij} = -\frac{\partial F_i}{\partial \dot{x}_j}$$

$$F_x = F_x(x, y, \dot{x}, \dot{y})$$

$$F_y = F_y(x, y, \dot{x}, \dot{y})$$

Since

$$-k_{xx} \equiv \frac{\partial F_x}{\partial x} \cong \frac{\Delta F_x}{\Delta x} = \frac{F_x(x+\Delta x, y, 0, 0) - F_x(x, y, 0, 0)}{\Delta x}$$

$$-k_{yx} \equiv \frac{\partial F_y}{\partial x} \cong \frac{\Delta F_y}{\Delta x} = \frac{F_y(x+\Delta x, y, 0, 0) - F_y(x, y, 0, 0)}{\Delta x}$$

$$-k_{xy} \equiv \frac{\partial F_x}{\partial y} \cong \frac{\Delta F_x}{\Delta y} = \frac{F_x(x, y+\Delta y, 0, 0) - F_x(x, y, 0, 0)}{\Delta y}$$

$$-K_{yy} \equiv \frac{\partial F_y}{\partial y} \cong \frac{\Delta F_y}{\Delta y} = \frac{F_y(x, y+\Delta y, 0, 0) - F_y(x, y, 0, 0)}{\Delta y}$$

$$-C_{xx} \equiv \frac{\partial F_x}{\partial \dot{x}} \cong \frac{\Delta F_y}{\Delta \dot{x}} = \frac{F_x(x, y, \Delta \dot{x}, 0) - F_x(x, y, 0, 0)}{\Delta \dot{x}}$$

$$-C_{yx} \equiv \frac{\partial F_y}{\partial \dot{x}} \cong \frac{\Delta F_y}{\Delta \dot{x}} = \frac{F_y(x, y, \Delta \dot{x}, 0) - F_y(x, y, 0, 0)}{\Delta \dot{x}}$$

$$-C_{xy} \equiv \frac{\partial F_x}{\partial \dot{y}} \cong \frac{\Delta F_x}{\Delta \dot{y}} = \frac{F_x(x, y, 0, \Delta \dot{y}) - F_x(x, y, 0, 0)}{\Delta \dot{y}}$$

$$-C_{yy} \equiv \frac{\partial F_y}{\partial y} \cong \frac{\Delta F_y}{\Delta \dot{y}} = \frac{F_y(x, y, 0, \Delta \dot{y}) - F_y(x, y, 0, 0)}{\Delta \dot{y}}$$

To comply with the calculations above the RLE needs to be solved five (5) slightly different ways for each operating point selected. The five (5) different solutions are listed as

1) $(x, y, 0, 0) \Rightarrow$ equilibrium condition
2) $(x+\Delta x, y, 0, 0) \Rightarrow$ x-displacement perturbation about equilibrium
3) $(x, y+\Delta y, 0, 0) \Rightarrow$ y displacement perturbation about equilibrium
4) $(x, y, \Delta \dot{x}, 0) \Rightarrow$ x velocity perturbation " "
5) $(x, y, 0, \Delta \dot{y}) \Rightarrow$ y velocity perturbation " "