with the Lagrange multiplier λ introduced to maintain the constraint $\phi \cdot \phi = 1$.

The Euler-Lagrange equations are given by

$$(1 - \partial_\mu \phi \cdot \partial^\mu \phi)\Box\phi - (\partial^\nu \phi \cdot \partial_\mu \partial_\nu \phi - \partial_\mu \phi \cdot \Box\phi)\partial^\mu \phi + (\partial^\mu \phi \cdot \partial^\nu \phi)\partial_\mu \partial_\nu \phi - \lambda\phi = 0\,, \tag{2.3}$$

where the Lagrange multiplier can be calculated by contracting (2.3) with ϕ and using the second derivative of the constraint,

$$\begin{aligned} \lambda = & \quad (1 - \partial_\mu \phi \cdot \partial^\mu \phi)\phi \cdot \Box\phi + (\partial^\mu \phi \cdot \partial^\nu \phi)(\phi \cdot \partial_\mu \partial_\nu \phi) \\ = & \; -(\partial_\mu \phi \cdot \partial_\nu \phi)(\partial^\mu \phi \cdot \partial^\nu \phi) - (1 - \partial_\mu \phi \cdot \partial^\mu \phi)\partial_\nu \phi \cdot \partial^\nu \phi. \end{aligned} \tag{2.4}$$

Denoting differentiation with respect to time as a dot, these equations can be recast as

$$M\ddot{\phi} - \alpha(\dot{\phi}, \partial_i \phi, \partial_i \dot{\phi}, \partial_i \partial_j \phi) - \lambda\phi = 0\,, \tag{2.5}$$

where the symmetric matrix M has elements

$$M_{ab} = (1 + \partial_j \phi \cdot \partial_j \phi)\delta_{ab} - \partial_j \phi_a \partial_j \phi_b\,, \tag{2.6}$$

and α is given by

$$\begin{aligned} \alpha = & (\dot{\phi} \cdot \partial_i \partial_i \phi - \partial_i \phi \cdot \partial_i \dot{\phi})\dot{\phi} + 2(\dot{\phi} \cdot \partial_i \phi)\partial_i \dot{\phi} - (\dot{\phi} \cdot \partial_i \dot{\phi})\partial_i \phi - \dot{\phi}^2 \partial_i \partial_i \phi \\ & + (\partial_i \phi \cdot \partial_i \partial_j \phi - \partial_j \phi \cdot \partial_i \partial_i \phi)\partial_j \phi + (1 + \partial_j \phi \cdot \partial_j \phi)\partial_i \partial_i \phi - (\partial_i \phi \cdot \partial_j \phi)\partial_i \partial_j \phi. \end{aligned} \tag{2.7}$$