

In the $SU(2)$ form, the equations of motion are cumbersome to handle numerically, so we convert to the notation of a non-linear sigma model (NLSM), which has Lagrangian,

$$\mathcal{L} = \partial_\mu \phi \cdot \partial^\mu \phi - \frac{1}{2} (\partial_\mu \phi \cdot \partial^\mu \phi)^2 + \frac{1}{2} (\partial_\mu \phi \cdot \partial_\nu \phi) (\partial^\mu \phi \cdot \partial^\nu \phi) + \lambda (\phi \cdot \phi - 1), \quad (2.2)$$